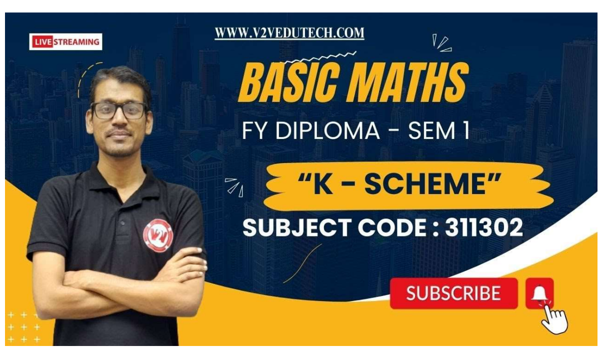
Some Important Lessons YouTube Lecture Links:

Lecture 1: Derivative: https://www.youtube.com/Derivative Lec 1
Lecture 2: Derivative: https://www.youtube.com/Derivative Lec 2
Lecture 3: Application of Derivative: https://www.youtube.com/Application of Derivative
Lecture 4: Function: https://www.youtube.com/Functions M1

Join Whtassp Group

K – Scheme : https://chat.whatsapp.com/B5tS6rgj5pp4lRFHAWbc3P



Mo - **9326050669 / 9372072139** **3 hrs**

Basic Mathematics (Fy Diploma Sem 1) **70 Marks**

K Scheme **Course code: 311302** **Prelim 2**

Q 1 Attempt any 5 **(10 Marks)**

a. Solve:
$\log(x+3)+\log(x-3)=10916$

b. Find area of triangle whose vertices are (4,5), (0,7) & (-1,1)

c. Without calculator find value of:
$\sec^2(3660^0)$

d. Mean and SD of distribution is 60 & 5 respectively find coefficient of variation.

e. Mean and SD of distribution is 60 & 5 respectively find coefficient of variation.

f. Find perp. distance of point (-3, 4) from line $4(x+2)=3(y-4)$

g. Find coefficient of range for 40, 52, 47, 28, 45, 36, 47, 50.

h. Find dy /dx if $y = (x^2+5)^7$

Q 2 Attempt any 3 **(12 Marks)**

a. If $A=\begin{bmatrix}2 & 1\\ 2 & 3\end{bmatrix}, B=\begin{bmatrix}1 & 2\\ 3 & -2\end{bmatrix}$
Whether AB is singular or non-singular matrix?

b. Show that $A^2 = I$, if

$$A=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$$

c. Solve using Cramer's rule
$v_1+v_2+v_3=9$
$v_1-v_2+v_3=3$
$v_1+v_2-v_3=1$

d. Solve using Matrix Inversion Method
$x+y+z=3$
$x+2y+3z=4$
$x+4y+9z=6$

Q 3 Attempt any 3 **(12 Marks)**

a. Resolve partial fraction
$\frac{9}{(x-1)(x+2)^2}$

b. Resolve partial fraction
$\frac{x^2-x+3}{(x-2)(x^2+1)}$

c. Resolve partial fraction
$\frac{x+3}{(x-1)(x+1)(x+5)}$

d. Resolve partial fraction
$\frac{x^2+1}{x^2-1}$

Q 4 Attempt any 3 **(12 Marks)**

a. Find angle between the lines
$y=5x+6 \;\&\; y=x$

b. Find equation of line passing through, $(3,-1)$ *and paralle to* x+2y-4=0

c. Solve without using calculator
$\sin(420)+\sin(-330).\cos(105)$

d. Prove that
$\tan^{-1}\left(\frac{1}{8}\right)+\tan^{-1}\left(\frac{1}{5}\right)=\tan^{-1}\left(\frac{1}{3}\right)$

Q 5 Attempt any 3 **(12 Marks)**

a. Find SD, variance & Coefficient of variance for following data:

Class	0-30	30-60	60-90	90-120
fi	10	20	30	40

b. Find range and coefficient of range for following data

CI	0-9	10-19	20-29	30-39	40-49
F	12	22	10	14	16

c. Divide 120 into two parts such a way that their product is maximum

d. Slope of tangent to curve curve $2y^3=ax^2+b$ at (1, -1) is same as slope of x + y = 0
Find a & b.

Q 6 Attempt any 2 **(12 Marks)**

a. Find radius of curvature to curve $y = e^x$ at x = 0

b. Find dy/dx if x = a (θ – sin θ) and y = a (1 – cos θ)

c. Find $\frac{dy}{dx}$ if $x^2 + y^2 + xy - y$ at (1,2)

d. $find\,\frac{dy}{dx}\,if$
$y=(sin^{-1}x)^x+(\cos x)^{si}$

Q 1

a) $\log(x+3) + \log(x-3) = 10196$

$\Rightarrow$

$\log\overset{a}{(x+3)} + \log\overset{b}{(x-3)} = 10196$

$\therefore \log[\overset{a}{(x+3)} \cdot \overset{b}{(x-3)}] = 10196$ ---- $\{\because \log a + \log b = \log(ab)\}$

$\therefore \log[x^2 - 3^2] = 10196$ ---- $\{(a-b)(a+b) = a^2 - b^2\}$

$\therefore \log(x^2 - 9) = 10196$

i.e $\log_e(x^2-9) = 10196$

$Y = A^X$ ——— exponential form

$X = \log_A Y$ ——— logarithmic form

Eq^n ① Comparing with logarithmic form

$X = 10196$

$A = e$

$Y = (x^2 - 9)$

Hence in terms of exponential form

$(x^2 - 9) = e^{10196}$

$x^2 = e^{10196} + 9$

$\therefore x = \sqrt{e^{10196} + 9}$ //

Q 1

b) Area of triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$(x_1, y_1) = (4, 5)$

$(x_2, y_2) = (0, 7)$

$(x_3, y_3) = (-1, 1)$

$$= \frac{1}{2}\begin{vmatrix} 4 & \boxed{5} & 1 \\ 0 & 7 & 1 \\ -1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[4\begin{vmatrix} 7 & 1 \\ 1 & 1 \end{vmatrix} - \boxed{5}\begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & 7 \\ -1 & 1 \end{vmatrix} \right]$$

$$= \frac{1}{2}\left\{ 4(7\times1 - 1\times1) - 5[0\times1 - 1\times(-1)] + 1[0\times1 - 7\times(-1)] \right\}$$

$$= \frac{1}{2}\{24 - 5 + 7\}$$

$$= \frac{1}{2}\times 26$$

$$= 13 \text{ sq.units}$$

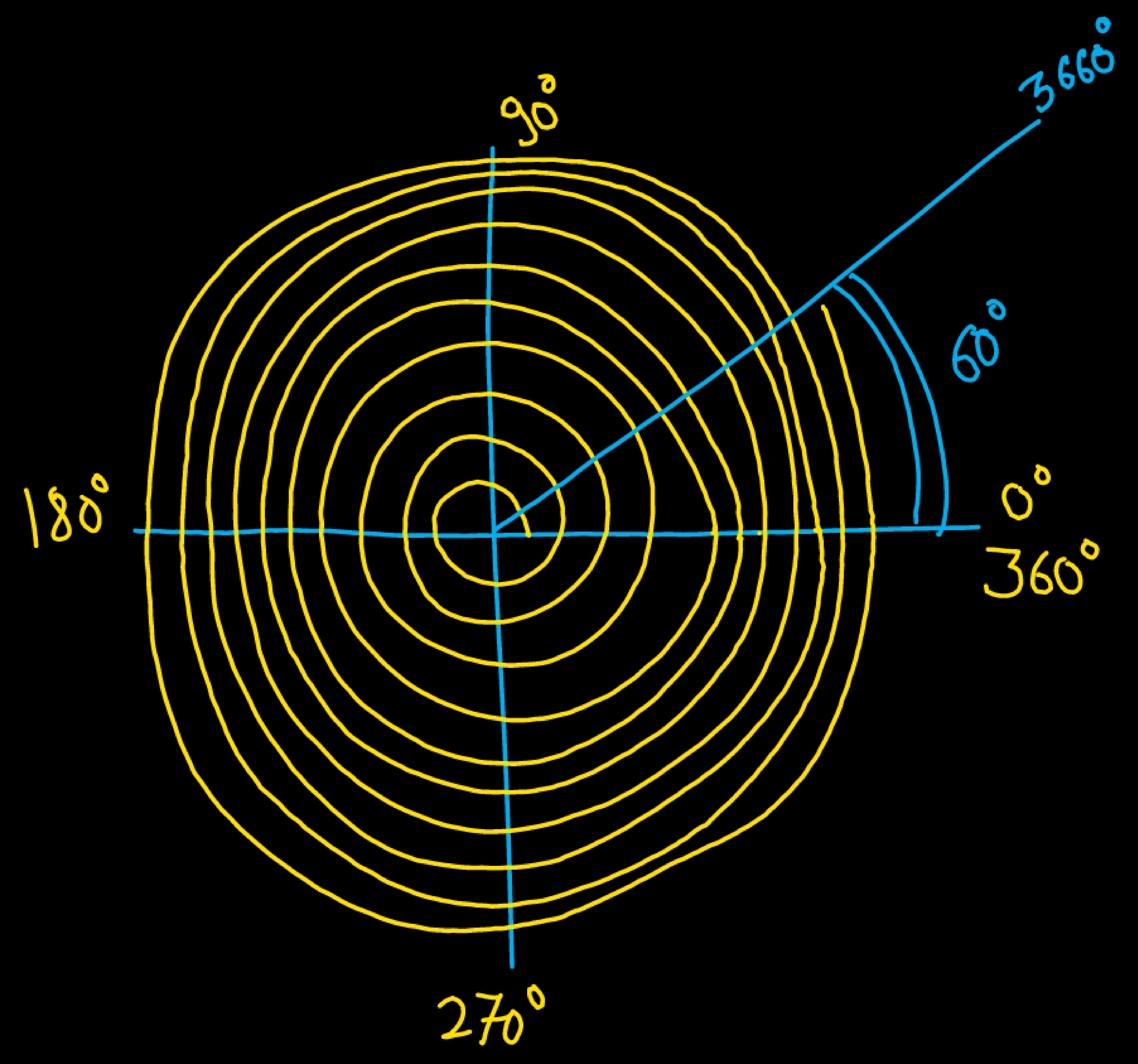
Q 1 c)

$$\sec^2(3660)$$
$$= [\sec(3660)]^2$$

$\sec(3660)$

$$\frac{3660}{90} = 40.667$$

40 $\rightarrow$ even $\longrightarrow$ self.



$$\therefore \sec(3660) = \sec(40 \times 90 + 60)$$
$$= + \sec(60)$$

positive since in Ist quadrant
all trignometric angles are positive

$$= +2$$

i.e $\sec(3660) = +2$

$$[\sec(3660)]^2 = (+2)^2$$
$$= +4$$

Q1

d)

Given $\Rightarrow$ Mean $= \bar{x} = 60$

SD $= \sigma = 5$

To find $\Rightarrow$

Coefficient of variation $= ?\ \%$

Solution $\Rightarrow$

$$\text{Coeff. of variation} = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{5}{60} \times 100$$

$$= 8.333\ \%$$

Q 1

f)

Given :- Point $= (x_1, y_1) = (-3, 4)$

Eq^n of line $\Rightarrow$ $4(x+2) = 3(y-4)$

To find :- Perpendicular distance of point from line $= d = ?$ units

Solution :-

$$d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}} \right|$$

eq^n of line $\Rightarrow$ $4(x+2) = 3(y-4)$

$4x + 8 = 3y - 12$

$\therefore 4x - 3y + 8 + 12 = 0$

$\therefore 4x - 3y + 20 = 0$ — Eq^n of st. line

$Ax + By + C = 0$ — Eq^n of st. line

$\therefore A = 4$, $B = -3$ & $C = 20$

& point $\Rightarrow (x_1, y_1) = (-3, 4)$

$$\therefore d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}} \right|$$

$$= \left| \frac{4\times(-3) + (-3)\times 4 + 20}{\sqrt{4^2 + (-3)^2}} \right|$$

$$= \left| \frac{-4}{\sqrt{25}} \right|$$

$$= |-0.8|$$

$d = 0.8$ units

Q1 g)

Given ⇒ Raw data

40, 52, 47, 28, 45, 36, 47, 50

To find ⇒ Coefficient of Range = ?

Solution ⇒

Range = U − L

U = Upper Value = 52

L = Lower Value = 36

∴ Range = U−L

= 52−36

= 16

∴ Coeff. of Range = $\frac{U-L}{U+L}$

$= \frac{52-36}{52+36}$

$= \frac{16}{88}$

= 0.1818

≃ 0.182 //

Q1 h) $y = (x^2+5)^7$

$\Rightarrow$ diff. w.r.t x

$$\frac{d}{dx}y = \frac{d}{dx}(x^2+5)^7$$

X

$$= \frac{d}{dx}X^7$$

$$= 7 \cdot X^{7-1} \cdot \frac{d}{dx}X$$

$$= 7(x^2+5)^6 \cdot \left[\frac{d}{dx}(x^2+5)\right]$$

$$= 7(x^2+5)^6 \left[\frac{d}{dx}x^2 + \frac{d}{dx}5\right]$$

$$= 7(x^2+5)^6 \left[2x^{2-1} + 0\right]$$

$$\frac{dy}{dx} = 7(x^2+5)^6 (2x^1)$$

Q2

a) $A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$

$$A \cdot B = \underset{2\times2}{\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}}$$

$$= \begin{bmatrix} 2\times1+1\times3 & 2\times2+1\times(-2) \\ 2\times1+3\times3 & 2\times2+3\times(-2) \end{bmatrix} \qquad \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$AB = \begin{bmatrix} 5 & 2 \\ 11 & -2 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 5 & 2 \\ 11 & -2 \end{vmatrix}$$

$$= 5\times(-2) - 11\times2$$

$$= -32$$

$$\neq 0$$

$\therefore$ [AB] is Non-singular Matrix

Q2 b)

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}_{3\times3} \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}_{3\times3}$$

$$= \begin{bmatrix} 0\times0+1\times4+(-1)\times3 & 0\times1+1\times(-3)+(-1)\times(-3) & 0\times(-1)+1\times4+(-1)\times4 \\ 4\times0+(-3)\times4+4\times(3) & 4\times1+(-3)\times(-3)+4\times(-3) & 4\times(-1)+(-3)\times4+4\times4 \\ 3\times0+(-3)\times4+4\times3 & 3\times1+(-3)\times(-3)+4\times(-3) & 3\times(-1)+(-3)\times4+4\times4 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{33} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Q2. c) $v_1 + v_2 + v_3 = 9$ —— ①

$v_1 - v_2 + v_3 = 3$ —— ②

$v_1 + v_2 - v_3 = 1$ —— ③

$\Rightarrow$

$$D = \begin{vmatrix} 1 & 1 & \boxed{1} \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} + \boxed{1}\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= 1[(-1)(-1) - 1\times1] - 1[1\times(-1) - 1\times1] + 1[1\times1 - 1\times(-1)]$$

$$= 0 - 1(-2) + 1(2)$$

$$D = 4$$

$$D_{v_1} = \begin{vmatrix} 9 & 1 & \boxed{1} \\ 3 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 9\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} + \boxed{1}\begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= 9[(-1)(-1) - 1\times1] - 1[3\times(-1) - 1\times1] + 1[3\times1 - 1\times(-1)]$$

$$= 9(1-1) - 1(-3-1) + 1[3-(-1)]$$

$$= 0 - 1(-4) + 1(3+1)$$

$$= 0 + 4 + 4$$

$$D_{v_1} = 8$$

$$D_{v_2} = \begin{vmatrix} 1 & 9 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 1\begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} - 9\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix}$$

$$= 1(-3-1) - 9(-1-1) + 1(1-3)$$

$$= 1(-4) - 9(-2) + (-2)$$

$$= -4 + 18 - 2$$

$$D_{v_2} = 12$$

$$D_{v_3} = \begin{vmatrix} 1 & 1 & 9 \\ 1 & -1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 1\begin{vmatrix} -1 & 3 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} + 9\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= 1(1-3) - 1(1-3) + 9[1-(-1)]$$

$$= 1(-4) - 1(-2) + 9(2)$$

$$= -4 + 2 + 18$$

$$D_{v_3} = 16$$

$\therefore$ By Cramer's Rule

$$v_1 = \frac{D_{v_1}}{D} = \frac{8}{4} = 2 \quad //$$

$$v_2 = \frac{D_{v_2}}{D} = \frac{12}{4} = 3 \quad //$$

$$v_3 = \frac{D_{v_3}}{D} = \frac{16}{4} = 4 \quad //$$

$$\therefore \boxed{v_1 = 2 \ ; \ v_2 = 3 \ \& \ v_3 = 4}$$

Q 5]

c] divide 120 into 2 parts

step ① forming equation of 'y'

120

x $120-x$

such that product is Max.

$\downarrow$

Multiplication

$$\therefore y = x(120-x)$$
$$y = 120x - x^2$$

Step ② : finding '$\frac{dy}{dx}$'

$$y = 120x - x^2$$

differentiating w.r.t x

$$\frac{d}{dx}y = \frac{d}{dx}(120x - x^2)$$

$$\frac{dy}{dx} = \frac{d}{dx}120x - \frac{d}{dx}x^2$$

$$\frac{dy}{dx} = 120\frac{d}{dx}x - 2x^{2-1}$$

$$\therefore \frac{dy}{dx} = 120.(1) - 2x$$

$$\therefore \frac{dy}{dx} = 120 - 2x$$

Step ③ find value of x if $\frac{dy}{dx} = 0$

$$\therefore \frac{dy}{dx} = 0 = 120 - 2x$$

$$\therefore 2x = 120$$

$$\therefore x = \frac{120}{2}$$

$$\therefore x = 60 \;//$$

Step ④ find $\frac{d^2y}{dx^2}$ to find Maxima or Minima

$$\because \frac{dy}{dx} = 120 - 2x$$

differentiating w.r.t 'x'

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(120 - 2x)$$

$$= \frac{d}{dx}120 - \frac{d}{dx}2x$$

$$= 0 - 2\frac{d}{dx}x$$

$$= 0 - 2 \times 1$$

$$\frac{d^2y}{dx^2} = -2 \;//$$

$\because \frac{d^2y}{dx^2}$ is negative

function is Maxima

Step ⑤ $\therefore$ function is Maxima

120

x $120-x$

$x = 60$ $\therefore 120 - 60 = 60$

$\therefore$ first part of '120' is 60

& Second part is $120 - x = 60$ //

Q 5)

d) Given ⇒ curve $2y^3 = ax^2 + b$

at Point $(x_1, y_1) = (1, -1)$

tangent slope is same as $x + y = 0$

<u>To find</u> ⇒ $a = ?$

$b = ?$

<u>Solution</u> ⇒

step 1 ⇒ finding slope of tangent

as per given data

slope of tangent is same as slope of line

∴ finding slope of line

$x + y = 0$

$Ax + By + C = 0$

$\therefore A = 1,\ B = 1,\ C = 0$

$\therefore$ slope of line $= -\frac{A}{B} = \frac{-1}{1} = -1$

$\therefore$ slope of tangent $= m_1 = -1$ // ——— (1)

step 2 finding slope of tangent

using eq^n of curve i.e $\frac{dy}{dx}$ at $(1, -1)$

Eq^n of curve $2y^3 = ax^2 + b$

differentiating w.r.t 'x'

$$\frac{d}{dx} 2y^3 = \frac{d}{dx}(ax^2 + b)$$

$$2 \cdot \frac{d}{dx} y^3 = \frac{d}{dx} ax^2 + \frac{d}{dx} b$$

$$2 \cdot \frac{d}{dx} x^3 = a \cdot \frac{d}{dx} x^2 + 0$$

$$2 \cdot [3x^{3-1}] \frac{d}{dx} x = a \cdot (2x)$$

$$2\left[3y^2 \cdot \frac{d}{dx} y\right] = 2ax$$

$$6y^2 \cdot \frac{dy}{dx} = 2ax$$

$$\therefore \frac{dy}{dx} = \frac{\cancel{2}ax}{\cancel{6}y^2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{3} \frac{ax}{y^2}$$

$$\left.\frac{dy}{dx}\right|_{at(1,-1)} = \frac{1}{3} \times \frac{a \times 1}{(-1)^2}$$

$$= \frac{1}{3} \times \frac{a \times 1}{1}$$

$$= \frac{1}{3} \times a$$

$$\left.\frac{dy}{dx}\right|_{at(1,-1)} = \frac{a}{3} \text{ ——— (1)}$$

step 3 Equating (1) & (2)

∵ both equation represents slope of tangent

$$\therefore \frac{a}{3} = -1$$

$$\therefore a = -1 \times 3$$

$$a = -3 \text{ //}$$

Step 4 finding value of 'b'

eq^n of curve $2y^3 = ax^2 + b$

& curve passing through $(1, -1)$

∴ Substituting value of $x = 1$ & $y = -1$

in eq^n of curve and $a = -3$

$$\therefore 2y^3 = ax^2 + b$$

$$2(-1)^3 = (-3) \times (1)^2 + b$$

$$2(-1) = (-3) \times 1 + b$$

$$-2 = -3 + b$$

$$-2 + 3 = b$$

$$\therefore 1 = b \text{ //}$$

$$\therefore \boxed{a = -3 \ \& \ b = 1} \text{ //}$$

Q 6

a) $y = e^x$ at $x=0$

find Radius of Curvature

$\Rightarrow$ step ① finding $\frac{dy}{dx}$

$y = e^x$

diff. w.r.t x

$\frac{d}{dx} y = \frac{d}{dx} e^x$

$\frac{dy}{dx} = e^x$

$\left.\frac{dy}{dx}\right|_{at\, x=0} = e^0$

$= 1$

step ② finding $\frac{d^2y}{dx^2}$

$\frac{dy}{dx} = e^x$

diff. w.r.t x

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx} e^x$

$\frac{d^2y}{dx^2} = e^x$

$\left.\frac{d^2y}{dx^2}\right|_{at\, x=0} = e^0$

$= 1$

step ③

Radius of Curvature $= \rho = \dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$

$= \dfrac{\left[1+(1)^2\right]^{1.5}}{1}$

$= \dfrac{[2]^{1.5}}{1}$

$= 2.8284$ units

$\rho \cong 2.828$ units

Q 6 b)

$x = a(\theta - \sin\theta)$ $\qquad$ $y = a(1-\cos\theta)$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$x = a(\theta - \sin\theta)$

diff. w.r.t θ

$$\frac{d}{d\theta}x = \frac{d}{d\theta}a(\theta - \sin\theta)$$

$$= a \cdot \frac{d}{d\theta}(\theta - \sin\theta)$$

$$= a\left[\frac{d}{d\theta}\theta - \frac{d}{d\theta}\sin\theta\right]$$

$$\frac{dx}{d\theta} = a\left[1 - \cos\theta\right]$$

$y = a(1-\cos\theta)$

diff. w.r.t θ

$$\frac{d}{d\theta}y = \frac{d}{d\theta}a(1-\cos\theta)$$

$$= a \cdot \frac{d}{d\theta}(1-\cos\theta)$$

$$= a\left[\frac{d}{d\theta}1 - \frac{d}{d\theta}\cos\theta\right]$$

$$= a\left[0 - (-\sin\theta)\right]$$

$$\frac{dy}{d\theta} = a \cdot \sin\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cancel{a} \cdot \sin\theta}{\cancel{a}(1-\cos\theta)} = \frac{\sin\theta}{1-\cos\theta}$$

Q 6

c) $x^2+y^2+xy-y=0$ considered. find $\frac{dy}{dx}$ at $(1,2)$

$\Rightarrow$ Implicite function $f(x,y)=0$

$$x^2+y^2+xy-y=0$$

diff. w.r.t x

$$\frac{d}{dx}x^2+\frac{d}{dx}y^2+\frac{d}{dx}\overset{u\ v}{xy}-\frac{d}{dx}y=0$$

$$2x+\frac{d}{dx}x^2+\left[u.\frac{d}{dx}v+v.\frac{d}{dx}u\right]-\frac{dy}{dx}=0$$

$$2x.\frac{d}{dx}x+\left[x.\frac{d}{dx}y+y.\frac{d}{dx}x\right]-\frac{dy}{dx}=-2x$$

$$2y.\frac{dy}{dx}+x.\frac{dy}{dx}+y.1-\frac{dy}{dx}=-2x$$

$$2y\frac{dy}{dx}+x.\frac{dy}{dx}-1\frac{dy}{dx}=-2x-y$$

$$\frac{dy}{dx}\left[2y+x-1\right]=-2x-y$$

$$\therefore \frac{dy}{dx}=\frac{-2x-y}{2y+x-1}$$

$$\left.\frac{dy}{dx}\right|_{at\ (1,2)}=\frac{-2\times1-2}{2\times2+1-1}$$

$$=\frac{-4}{4}$$

$$=-1$$

Q 6)

d) $y = (\sin^{-1}x)^{x} + (\cos x)^{\sin x}$

$y = U + V$

differentiating w.r.t 'x'

$\frac{d}{dx} y = \frac{d}{dx}(U+V)$

$\frac{dy}{dx} = \frac{d}{dx}U + \frac{d}{dx}V$

$\because U = (\sin^{-1}x)^{x}$ $\quad (\text{variable})^{\text{variable}}$ i.e $(\text{variable})^{\text{variable}}$

$\therefore$ Taking log on both sides

$\log U = \log(\sin^{-1}x)^{x}$

$\log U = x \cdot \log(\sin^{-1}x)$

differentiating w.r.t 'x'

$\frac{d}{dx}\log \overset{x_1}{U} = \frac{d}{dx}\, \overset{u}{x} \cdot \overset{v}{\log(\sin^{-1}x)}$

$\frac{d}{dx}\log x_1 = u \cdot \frac{d}{dx}v + v \cdot \frac{d}{dx}u$

$\frac{1}{x_1} \cdot \frac{d}{dx} x_1 = x \frac{d}{dx} \log(\overset{x}{\sin^{-1}x}) + \log(\sin^{-1}x) \frac{d}{dx} x$

$\frac{1}{U} \cdot \frac{d}{dx} U = x \cdot \frac{d}{dx} \log X + \log(\sin^{-1}x) \cdot 1$

$= x \cdot \frac{1}{X} \cdot \frac{d}{dx} X + \log(\sin^{-1}x)$

$= x \frac{1}{\sin^{-1}x} \cdot \frac{d}{dx} \sin^{-1}x + \log(\sin^{-1}x)$

$\frac{1}{U} \cdot \frac{d}{dx} U = \frac{x}{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}} + \log(\sin^{-1}x)$

$\frac{d}{dx} U = U\left[\frac{x}{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}} + \log(\sin^{-1}x)\right]$

$\frac{d}{dx} U = (\sin^{-1}x)^{x}\left[\frac{x}{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}} + \log(\sin^{-1}x)\right]$

$\because V = (\cos x)^{\sin x}$

$\therefore$ Taking log on bothsides

$\log V = \log(\cos x)^{\sin x}$

$\log V = \sin x \cdot \log(\cos x)$

differentiating w.r.t 'x'

$\frac{d}{dx}\log \overset{x_1}{V} = \frac{d}{dx}\, \overset{u}{\sin x} \cdot \overset{v}{\log(\cos x)}$

$\frac{1}{x_1} \frac{d}{dx} x_1 = u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u$

$\frac{1}{V} \cdot \frac{d}{dx} V = \sin x \cdot \frac{d}{dx} \log(\overset{x}{\cos x}) + \log(\cos x) \frac{d}{dx} \sin x$

$= \sin x \cdot \frac{d}{dx} \log X + \log(\cos x) \cdot \cos x$

$= \sin x \cdot \frac{1}{X} \cdot \frac{d}{dx} X + \cos x \cdot \log(\cos x)$

$= \sin x \cdot \frac{1}{\cos x} \cdot \frac{d}{dx} \cos x + \cos x \cdot \log(\cos x)$

$= \frac{\sin x}{\cos x} \cdot (-\sin x) + \cos x \cdot \log(\cos x)$

$\frac{1}{V} \cdot \frac{d}{dx} V = \tan x \cdot (-\sin x) + \cos x \cdot \log(\cos x)$

$\frac{d}{dx} V = V\left[\tan x \cdot (-\sin x) + \cos x \cdot \log(\cos x)\right]$

$\frac{d}{dx} V = (\cos x)^{\sin x}\left[\tan x \cdot (-\sin x) + \cos x \cdot \log(\cos x)\right]$

$\therefore \frac{dy}{dx} = \frac{d}{dx} U + \frac{d}{dx} V$

$= (\sin^{-1}x)^{x}\left[\frac{x}{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}} + \log(\sin^{-1}x)\right]$

$+ (\cos x)^{\sin x}\left[\tan x \cdot (-\sin x) + \cos x \cdot \log(\cos x)\right]$ //